□ Class 12 Mathematics – Chapter: Determinants

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1. Introduction

- Determinants are scalar values computed from square matrices.
- They provide useful properties about the matrix such as invertibility.

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2. Definition

- Determinant of a 2x2 matrix

  det□ [abcd]=ad−bc\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bcdet[acbd]=ad−bc
- Determinant of 3x3 and higher matrices is calculated using expansion by minors or cofactors.

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3. Minor and Cofactor

- **Minor:** Determinant of the submatrix obtained by deleting one row and one column.
- **Cofactor:**

  Cij=(−1)i+jMijC_{ij} = (-1)^{i+j} M_{ij}Cij=(−1)i+jMij

  where MijM_{ij}Mij is the minor corresponding to element aija_{ij}aij.

4. Properties of Determinants

- Interchanging two rows changes the sign of the determinant.
- If two rows are identical, determinant is zero.
- Determinant of identity matrix is 1.
- Multiplying a row by a scalar multiplies determinant by that scalar.
- Determinant of a product of matrices equals product of determinants.
- Adding a multiple of one row to another row does not change determinant.

5. Expansion of Determinants

- Can expand along any row or column.
- Determinant of 3x3 matrix A=[aij]A = [a_{ij}]A=[aij]:

  det⁡(A)=a11C11+a12C12+a13C13\det(A)= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}
  det(A)=a11C11+a12C12+a13C13

6. Applications

- Checking invertibility of matrices.
- Solving systems of linear equations (Cramer’s Rule).
- Area and volume calculations.

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7. Cramer’s Rule

- For system AX=BAX = BAX=B,

  xi=det⁡ Aidet⁡ Ax_i = \frac{\det A_i}{\det A}xi=detAdetAi

  where AiA_iAi is matrix AAA with ithi^{th}ith column replaced by BBB.

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8. Exam Tips

- Memorize properties and formulas.
- Practice expansion by minors and cofactors.
- Solve problems using Cramer’s rule.
- Understand effect of row operations on determinants.